Why Some Disaster Insurance Does not Exist
Yueyun(Bill) Chen1 and Iskandar S. Hamwi2

Abstract
The failure of some disaster insurance market has been a very serious problem. This paper focuses on why disaster reinsurance fails and how that will affect the availability of primary disaster insurance. The insurer’s unexpected costs are added to the expected costs associated with the insured event to illustrate the necessary conditions for the existence of disaster insurance and reinsurance. Particularly, investors’ negative response to an insurer’s huge, disaster-related liability exposures may lead to availability problem unless the insurer’s value losses in the financial market because of these exposures can be minimized. A large insurer may be more likely to withdraw from underwriting disaster insurance. Three different pricing schemes for disaster reinsurance contracts are investigated. The one which is based on the Option Pricing Theory is rejected because it leads to market failure.

Keywords: Disaster insurance, unexpected cost, options, asset value losses, fair price.

I. Introduction
Availability of disaster insurance and reinsurance has been a serious problem in recent years (Cummins et al., 2002, Froot, 1997, 1999, Kunreuther, 2006). For instance, following the big earthquake in the Bay area in San Francisco, California, many residents in that area found that they can no longer buy coverage of earthquake insurance from their original insurers. Following Hurricane Andrew, many residents in the eastern coast of the United States were declined disaster insurance coverage by most insurance companies. Recently, following the huge Katrina disaster in New Orleans, most insurers cancelled their offering of the relevant insurance coverage to the people in that area.

Several previous studies indicate that the unavailability of reinsurance is one of the major reasons which caused the primary insurers’ unwillingness to continue their business. For instance, Berger, Cummins and Tennyson (1992) show that the unavailability of reinsurance coverage contributed to the crisis of liability insurance. Hershbarger (1994) demonstrates that inaccessibility of disaster reinsurance leads to the unavailability of disaster insurance.

Conventional explanations for the unavailability of insurance or reinsurance include non-diversification of risk, adverse selection, and moral hazard (Borch,1990; Bum & Schlesinger, 2005). Given the nature of disaster insurance, these three factors may account, to some extent, for the non-existence of insurance or reinsurance, but they cannot fully explain the whole picture. The huge unexpected costs associated with disaster insurance significantly contributed to the failure of the disaster insurance market.

Such huge unexpected costs include the insurer’s extra market value loss associated with the extra decrease of its security (stocks and/or bonds) prices (Lamb, 1995, 1996)1, decrease of demand for its business (some consumers may worry about the insurer’s solvency so they refuse to buy coverage from the insurer) (Brown & Hoyt, 2000), unexpected and extra reimbursement to insureds and other factors such as extra payment to its employees working overtime and possible lawsuits. The insurer’s security price would likely decreases dramatically because investors would suddenly realize the firm’s

1 Department of Business Administration, University of the West, Rosemead, CA 1770; E-mail: billchen@uwest.edu
2 University of Southern Mississippi, Southern Station, Box 5076, Hattiesburg, MS 39406, E-mail: iskandar.hamwi@usm.edu
higher risk of its business. One useful indicator of the firm’s higher risk is the lowering of its financial rating by the professional financial rating firms.

Unexpected costs affect the decision of insurers as to whether or not to be engaged in the market competition. Aware of potential unexpected costs, some insurance firms may not want to do business in the area of disaster insurance. This may well explain why many primary insurers and reinsurers withdrew from the disaster insurance market.

Regarding the pricing of reinsurance, prior studies assume that the reinsurance market is perfectly competitive. In this paper, based on different conditions of insurance markets, three different pricing schemes for reinsurance are introduced. In addition, the necessary conditions for the existence of the reinsurance market and, therefore, the existence of the primary insurance market are given. Especially, the current research shows that both the primary and the reinsurance markets may fail under a price derived from the Option Pricing Theory while these markets can exist otherwise.

Pricing of reinsurance has been widely studied. Borch (1974) uses equilibrium theory to derive fair pricing for a reinsurance contract. Doherty and Garven (1986) give the formula of reinsurance premiums by applying the Option Pricing Theory. Venter (1993) values the reinsurance contract by using the Arbitrage Pricing Theory. In this paper, a reinsurance contract is considered as a put option to the primary insurer with the price of the reinsurance contract being subsequently determined under three different conditions of the reinsurance market -- a market monopolized either by the primary insurer or by the reinsurer and a market where the competitive conditions prevail. Consequently, three different pricing schemes for the reinsurance contract are given--the minimum one, the maximum one, and the fair one.

The approach followed in this paper in addressing the problem of pricing of reinsurance differs from all of those suggested by previous studies, including that of Doherty and Garven(1986) and Chen, et al. (2001). The unexpected losses as well as expected losses to insurance firms resulting from the occurrence of the insured event are considered in price determination in this study. The expected losses include the expected reimbursement to the insureds and regular administrative expenses of the insurer.

The rest of the paper is organized as follows: the next section explores different pricing schemes of the reinsurance contract and gives conditions for the existence of reinsurance. Section 3 gives a numerical example, which shows how both primary insurers and reinsurers may benefit by cooperating with each other. Section 4 demonstrates the effects of sizes of firms, unexpected costs and the reinsurance premiums on the availability of insurance. The last section provides some concluding remarks.

II. Options and the Value of Reinsurance

2.1. The Choice of Strategies and the Value of Reinsurance

Suppose there are two strategies for the primary insurer. Under strategy one the insurer’s net asset takes a random value X₁ with a distribution F₁ while under strategy two, the firm’s net asset has a random value X₂ with the distribution F₂, i. e. X₁ ~ F₁(α₁, σ₁) and X₂ ~ F₂(α₂, σ₂); where α₁ and σ₁ are the mean and standard deviation of X₁, respectively; and α₂ and σ₂ are the mean and standard deviation of X₂. In addition, α₁ < α₂, and σ₁ < σ₂. Min. (X₂) < 0 < Min(X₁). In other words, the primary insurer will have a higher value on average under strategy two, but its variance will be higher also. Moreover, the insurer may face the risk of bankruptcy under strategy two.

Assume that the primary insurer will not choose strategy two without reinsurance either because it does not want to bear the risk associated with this strategy, or because regulation prohibits the insurer from doing so.
Now, suppose that a reinsurance firm offers an option to the primary insurer. Let Ex be the exercise price and Op be the price of the option, Ex and Op ≥ 0. Then, one can illustrate what is the price of the option and under what conditions both the primary insurer and the reinsurer are willing to share risks and benefits.

The expected net benefit for the primary insurer from taking strategy two instead of one is \(\Delta = E(X_2 \mid X_2 \geq Ex) + Ex \cdot \text{Pr}(X_2 < Ex) - E(X_1) - Op\). On the other hand, the expected payment which the reinsurer encounters is \(P_m = E[\text{Max}(Ex - X_2, 0)] = Ex \cdot \text{Pr}(X_2 < Ex) - E(X_2 \mid X_2 < Ex)\).

Then, the necessary condition for the existence of reinsurance is \(Op \geq P_m\) and \(\Delta \geq 0\). Consequently, one has

1. \(E(X_2 \mid X_2 \geq Ex) + Ex \cdot \text{Pr}(X_2 < Ex) - E(X_1) - Op \geq 0\);
2. \(Op \geq Ex \cdot \text{Pr}(X_2 < Ex) - E(X_2 \mid X_2 < Ex)\).

Denote \(\Delta_1 = E(X_2) - E(X_1)\) and \(\Delta_2 = Op - Ex \cdot \text{Pr}(X_2 < Ex) + E(X_2 \mid X_2 < Ex)\). Thus, \(\Delta = \Delta_1 - \Delta_2\). From (1) and (2), one has \(\Delta_1 = E(X_2) - E(X_1) \geq 0\).

In a reinsurance market monopolized by the primary insurer, the reinsurer earns zero profit, i.e. \(\Delta_2 = 0\) or \(Op = Ex \cdot \text{Pr}(X_2 < Ex) - E(X_2 \mid X_2 < Ex)\). On the other hand, being a monopoly in the market, the reinsurer can seize all profits as much as it can, so \(\Delta = 0\), i.e. \(Op = Ex \cdot \text{Pr}(X_2 < Ex) + E(X_2 \mid X_2 \geq Ex) - E(X_1)\), denoted by \(P_{\text{min}}\), is the minimum premium that the reinsurer requires; and \(Op = E(X_2 \mid X_2 \geq Ex) + Ex \cdot \text{Pr}(X_2 < Ex) - E(X_1)\), denoted by \(P_{\text{max}}\), is the maximum premium that the primary insurer is willing to pay for reinsurance. Let \(O'p\) be the actual price of reinsurance, then the necessary conditions for the existence of reinsurance are \(P_{\text{max}} \geq Op \geq P_{\text{min}}\).

In addition, one can use the option pricing theory to derive a fair price for the option, where the exercise price is Ex and the present value of the asset is \(E(X_1)\). Let \(P_{\text{fair}}\) be the fair pricing from OPT; one has \(P_{\text{fair}} = E\{\text{Max}(Ex - X_2, 0)\}\), where E is the expectation operator. Thus, one has three different price schemes, \(P_{\text{min}}\), \(P_{\text{max}}\) and \(P_{\text{fair}}\). The necessary condition for the existence of reinsurance and therefore insurance is that \(P_{\text{min}} \leq P_{\text{fair}} \leq P_{\text{max}}\). When the reinsurance is priced according to \(P_{\text{fair}}\), the necessary conditions for the existence of both primary and reinsurance markets are \(P_{\text{min}} \leq P_{\text{fair}} \leq P_{\text{max}}\).

Furthermore, one may consider the reinsurer's asset losses from the adverse event of the disaster. Let \(L_R\) be the asset value losses to the reinsurer when the reinsurer encounters the liability exposure, then the expected cost to the reinsurer is

\[
P_m = Ex \cdot \text{Pr}(X_2 < Ex) - E(X_2 \mid X_2 < Ex) + L_R \cdot \text{Pr}(X_2 < Ex)
\]

Now, the necessary conditions for the existence of reinsurance become

1. \(\Delta = E(X_2 \mid X_2 \geq Ex) + Ex \cdot \text{Pr}(X_2 < Ex) - E(X_1) - Op \geq 0\);
2. \(\Delta_2 = Op - [Ex + L_R] \cdot \text{Pr}(X_2 < Ex) - E(X_2 \mid X_2 < Ex) \geq 0\);

Satisfying condition 1 will motivate the primary insurer to underwrite policies while having condition 2 will make the reinsurer have an incentive to participate in the disaster insurance.

2.2. Two-State Options and Pricing of Reinsurance

Suppose that the primary insurer has net assets \(A_p\) without underwriting disaster insurance. Assume further that, disaster insurance has a loss distribution that has total claim indemnities of \(L\) with a probability of \(q\).
Assume that the default rate of the primary insurer’s asset is $d_p$ when the insured event occurs and that the total premiums earned by the insurer are $P$. Then, the primary insurer’s contingent assets have the following distribution after participating in the disaster insurance scheme:

$$
(1-d_p) A_p + P - L
$$

The expected net assets to the primary insurer after underwriting the disaster insurer are $(1 - q d_p) A_p + (P - q L)$. The primary insurer may participate in issuing disaster insurance policies when $(1 - q d_p) A_p + (P - q L) > 0$ or $q d_p A_p + (P - q L) > 0$ (2.1)

From (2.1), one has $P > q L + q d_p A_p$. In other words, the premium that the insurer charges is at least as large as the expected payments, $q L$, plus the expected indirect losses, $q d_p A_p$.

The insurer’s probability of insolvency assuming coverage for the disaster insurance is $q$ which, if it is too high to satisfy certain regulatory requirements, will force the primary insurer to cede some premiums in order to satisfy solvency regulation.

Assume that the reinsurer has net assets $A_R$. The form of reinsurance takes excess of loss. Let $Ex$ and $Op$, be respectively, the exercise price and option price (called the premium of reinsurance$^4)$. In other words, if $(1 - d_p) A_p + (P - L) - Op < Ex$, the reinsurer will compensate the primary insurer. Also, assume that the default rate of the reinsurer’s asset is $d_R$, when the reinsurer needs to pay claims to the primary insurer. Thus, the expected costs to the reinsurer are $P_{m} = (Ex + d_R A_R) q - E(X_2 | X_2 < Ex) = q \{Ex + d_R A_R - [(1 - d_p) A_p + (P - L) - Op]\}$. The reinsurer has an incentive to do business only when $Op \geq P_{m}$.

Now, with reinsurance, the primary insurer’s expected net assets are $(1-q) \{(1-d_p) A_p + (P - L) - Op\} + q Ex$. Thus, the net benefit for the insurer by participating in the disaster insurance business is

$$
(1-q)(1-d_p) A_p + (P - L) - Op + q EX - A_p
$$

Thus, the necessary conditions for the existence of disaster insurance are

1. $\Delta = (1-q)(1-d_p) A_p + (P - L) - Op + q EX - A_p \geq 0$;
2. $\Delta_2 = Op - q \{Ex + d_R A_R - (1-d_p) A_p + (P - L) + Op\} \geq 0$;

When the primary insurer has the monopoly power over the reinsurer, one has $\Delta_2 = 0$. Hence,

$$
P_{min} = Op = \{q/(1-q)\}[Ex + d_R A_R - (1-d_p) A_p - P + L].
$$

On the other hand, when the reinsurance market is monopolized by the reinsurer, one has $\Delta = 0$, i.e.

$$
P_{max} = Op = P - \{q/(1-q)\} A_p + [q/(1-q)] Ex
$$

Moreover, one can derive the price of the reinsurance using the option pricing theory. Once again, let $Ex$ be the exercise price of the option and let $p$ be the hedging probability. Notice that $A_p$ is the present value of the primary insurer without option. Then, one has

$$
p \{ (1-d_p) A_p + P - L \} + (1-p)(A_p + P) = A_p
$$

One can solve for $p$ from the above equation and has

$$
p = [P/L + d_p A_p]
$$

Then, the expected value of the option is $p \{Ex - (1-d_p) A_p - P + L\}$. In other words, the fair price of the option is

$$
P_{fair} = Op = [P/L + d_p A_p] [Ex - (1-d_p) A_p - P + L] + q d_R A_R
$$
In (2.7), the second term $q \, dR_R$, is the unexpected costs to the reinsurer. One may consider it as the transaction costs to the firm. Thus, one may view the first term in (2.7) as the fair price without transaction costs.

Now, one has three different price schemes for the reinsurance contract. The minimum price $P_{\text{min}}$, the maximum one $P_{\text{max}}$, and the fair one $P_{\text{fair}}$. The necessary condition for the existence of the disaster reinsurance and insurance, as stated before, is that $P_{\text{min}} \leq P_{\text{max}}$. In addition, when the reinsurance contract is priced with $P_{\text{fair}}$, the necessary conditions will be $P_{\text{min}} \leq P_{\text{fair}} \leq P_{\text{max}}$.

### III. Numerical Example

Assume that the primary insurer has a net asset value of $100$ million. If it underwrites policies in a line like disaster insurance, it will, say, encounter potential direct losses of $300$ million with a probability 0.20. In addition, when the insured disaster occurs, assume that the insurer’s assets are defaulted in the financial market and the default rate is 10%. To avoid its bankruptcy, without reinsurance the primary insurer must charge premiums totaling at least $210$ million. Notice that the expected claim costs are only $60$ million (0.20 * $300$ million) and the expected unexpected costs associated with the disaster are $2$ million (0.20 * 10% * $100$ million). Therefore, the insurance premiums could be as low as $62$ million. Suppose that consumers are willing to be insured when the premiums they pay don’t exceed $80$ million. However, the primary insurer can not take advantage of this business due to solvency regulation which considers the insurer’s probability of insolvency of 20% when engaged in disaster insurance to be too high.

Suppose that a reinsurance firm has net assets of $300$ million. The default rate of the reinsurer’s assets is 5% when the reinsurer is engaged in disaster reinsurance. Let $E_x$ be the exercise price the reinsurer offers to the primary insurer and $O_p$, be the price of the option or the premium of reinsurance. From the discussion in section 2.2, one knows that the minimum premium of reinsurance to be:

$$P_{\text{min}} = \frac{.2}{.8} [E_x + 5\% \times 300 - .9 \times 100 - P + 300] = .25 [E_x - P] + 56.25$$

On the other hand, the maximum premium is

$$P_{\text{max}} = P - .25 \times 100 + .25 E_x = P + .25 E_x - 25$$

And, the fair premium derived from the OPT is

$$P_{\text{fair}} = \frac{P}{L + dpA_P} [E_x - (l-d_p)A_P - (P - L)] + q \, dR_R$$

To have $P_{\text{max}} \geq P_{\text{min}}$ one needs $P + .25 E_x - 25 \geq .25 [E_x - P] + 56.25$; or $P \geq 65$. In other words, the premiums the policyholders pay must be at least $65$ million dollars, which are the total expected costs consisting of the expected direct losses of $60$ million, the expected unexpected costs for the primary insurer of $2$ million, and the expected unexpected costs for the reinsurer of $3$ million.

In case $E_x = 0$, one has

$$P_{\text{min}} = 56.25 - .25 P; \quad P_{\text{max}} = P - 25; \quad \text{and} \quad P_{\text{fair}} = 3 + P(210 - P)/310.$$  

When $E_x = 100$, one has

$$P_{\text{min}} = 81.25 - .25 P; \quad P_{\text{max}} = P; \quad \text{and} \quad P_{\text{fair}} = 3 + P(310 - P)/310.$$  

Furthermore, one can explore the pricing of reinsurance given the insurance premiums paid by the policyholders. Suppose that $P = 65$, then one has

$$P_{\text{min}} = P_{\text{max}} = .25 E_x + 40; \quad \text{and} \quad P_{\text{fair}} = 3 + 65/310 (E_x + 145) = 33.40 + .21E_x$$

So, $P_{\text{fair}} < P_{\text{min}}$. In other words, if the reinsurance premium is set to equal the $P_{\text{fair}}$ determined from the OPT, the reinsurer will suffer a net loss and, therefore, it will have no incentive to do business.
In fact, it can be verified that $P$ must equal to 79.28 to guarantee that $P_{\text{fair}} \geq P_{\text{min}}$ for $E_x = 0$. If $P = 79.28$ and $E_x = 0$, then $P_{\text{min}} = P_{\text{fair}} = 36.43$, and $P_{\text{max}} = 54.28$.

It is interesting to notice that a fair reinsurance premium derived using the OPT leads to insurance market failure.

IV. Unexpected Costs, Pricing of Reinsurance and Availability of Insurance

The previous section indicates that the reinsurance market and thus the insurance market may fail because of the unexpected costs and pricing of reinsurance.

First, unexpected costs may lead to the failure of the insurance markets because of the following reasons. If the insurance premiums the policyholders need to pay include the expected losses plus unexpected losses, when there are huge unexpected losses from the financial market associated with the insured events, the loading factor in the premiums charged to policyholders will be so big that no consumers would want to purchase insurance. On the other hand, when insurance firms can not, because of rate regulation, adjust their premiums to cover the unexpected costs, no firm will have the incentive to underwrite disaster insurance policies.

Secondly, the pricing of reinsurance using the OPT may cause the failure of the reinsurance market. Notice that the OPT is based on the assumption that the reinsurance market is perfectly competitive, or there is no arbitrage. If so, the price of the option is whatever its value will be to the primary insurer. As a result, the price derived from the OPT does not guarantee the existence of the reinsurance market because it ignores the profit incentives of the reinsurance firms.

The example demonstrated in section 3 shows that one may have $P_{\text{fair}} < P_{\text{min}}$; in other words, a reinsurer may not be compensated enough when the price is set up according to $P_{\text{fair}}$. On the other hand, one can find a case with $P_{\text{fair}} > P_{\text{max}}$. This can happen when the reinsurer suffers huge indirect losses denoted by $D_R A_R^4$.

In addition, the size of the firm affects the availability of insurance. In order to underwrite policies like the ones in disaster insurance, a firm must have enough assets. However, a big firm may not have any competitive advantages due to the unexpected costs. To see this, let two firms have assets $A_1$ and $A_2$, respectively, with $A_1 > A_2$. And let $d_1$ and $d_2$ be the default rate of their assets when the insured event occurs. Thus, the total defaulted assets to the big firm are $d_1 A_1$ and the total defaulted assets to the small firm are $d_2 A_2$. The big firm will have its advantage or low unexpected costs only when $d_1 A_1 < d_2 A_2$, or $d_1 < d_2 A_2/A_1$. Assume that the big firm has twice the assets of the small firm. Then, the big firm has lower unexpected costs only when its default rate is less than half of that of the small firm.

V. Conclusions

By making insurers’ cost include unexpected costs as well as expected costs associated with the insured event, this paper illustrates conditions for the existence of disaster insurance and reinsurance. The study particularly demonstrates that investors’ negative response to an insurer’s huge liability exposures related to the disaster could prohibit that insurer from continuing its coverage to the policyholders.

A large insurer is not necessarily more willing to engage in underwriting disaster insurance unless it can minimize its value losses in the financial market caused by the liability exposures.

Moreover, the paper gives three different pricing schemes for reinsurance- a maximum price, a minimum price, and a fair price, based on certain market conditions. The study demonstrates that a fair reinsurance premium derived from the OPT may lead the failure of all insurance markets.
Endnotes

1. Even if the insurer does not have the stocks/bonds publicly traded, the firm may still bear extra unexpected financial loss since the interest rate the bank charges to the insurer will increase given its higher financial risk. More generally, the required rates of returns of all types of the investors (debt-holders and stock-holders) will be higher when the insurer has a higher risk, so the cost of the capital to the insurer will be higher consequently.

2. An insurer will be able to bear such a disaster insurance risk when it can raise extra money from the financial market. Here, the assumption is that raising needed money is too costly for the insurer in the short time.

3. Each insurer is required to have a minimum surplus ratio by the state’s solvency regulation.

4. The relation between the stop loss and the put option is straightforward. Let $L$ be the stop loss point, i.e., the reinsurer will compensate the primary insurer for all extra losses when the primary insurer’s loss exceeds $L$; then, $E_x = (1-d_p)A_p + P - O_p - L$. Notice that $D_R A_R$ occurs in the $P_{fair}$ but it does not occur in the $P_{max}$.

References


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